Sound Propagation in Multistage Axial Flow Turbomachines

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A method is described for determining the sound propagation in multistage turbomachines. It is applicable to turbomachines which have nonuniform annular ducts carrying a mean flow with a swirling component. Comparison with measurements and the results from other theories show good compatibility. A computational study illustrates the influence of the swirling flow component and other parameters on the sound propagation. Further, numerical results are presented which indicate a strong acoustic coupling of the cascades.

Nomenclature

A,B,C,D	= elements of four-pole matrix
a	= amplitude factor of velocity potential
b	= axial length of a duct element
c	= velocity of sound
F	= area of duct cross section
f	= frequency
f_c	= cutoff frequency of an exponential horn
g	= arbitrary function
h	= height of an annulus
i	$=(-1)^{1/2}$
k	= sound propagation constant
k_0	= wavenumber $=\omega/c$
l	= chord length
Ma	= Mach number
m,n	= order of the circumferential and radial modes
p	= sound pressure
R	= pressure reflection coefficient
r	= mean radius of annulus
t	= time
T	= pressure transmission coefficient
TL	= power transmission loss
U,V,W	= total flow velocities along x,y,z
u,v,w	= acoustic velocities along x,y,z
$\bar{u},\bar{v},\bar{w}$	= steady flow velocities along x,y,z
x,y,z	= annular duct coordinates in axial, circumferential, and radial direction, see Fig. 3
Z	= acoustic impedance
Z_0	= acoustic impedance with reflection-free sound propagation
α,β	= components of axial propagation constant
ΔSPL	= sound pressure level difference
θ	= specific resistance
λ	= wavelength
P	= total density
$\bar{\rho},\rho$	= steady and acoustic density
ϕ	= acoustic velocity potential
χ	= specific reactance
ω	= circular frequency

Superscripts

$(\ )^+$	= downstream propagation
$(\ )^-$	= upstream propagation

Subscripts

j	= quantity in plane j
$1j$	= sound propagation between planes 1 and j
x,y,z	= x, y, or z component
1	= inlet of a duct element or cascade
2	= outlet of a duct element or cascade

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Introduction

In modern aircraft engines with high bypass ratios, a major portion of the sound emitted comes from the interior of the engine. The main sources of this sound are the fan, turbine, combustion chamber, and the compressor. The sound from these components gains access to the outside predominantly through the ducts or through the intake and nozzles. Depending on the location of the source, the sound must propagate through a larger or smaller number of cascades, where a considerable portion of the source's original sound power is lost as a result of reflection and attenuation. In the case of the combustion chamber the attenuation, for example by the high- and low-pressure turbines, including the nozzle, amounts to 10-15 PWLdB. Accordingly, it is imperative to bear these propagation losses in mind in the acoustic design of engines or when estimating the sound emission of engines.

Sound propagation investigations with turbomachinery have, so far, largely concentrated on propagation via bladeless ducts[1] and only to a very limited extent on that via ducts containing cascades. There are very few results available of experimental investigations into propagation through cascades.[2] Moreover, theoretical investigations into this subject have so far concentrated on the passage of sound through individual cascades.[3-7] Only Amiet[8] and Matta[9] provide theories concerning a series of cascades arranged one behind the other, and even here Amiet confines his research to propagation via two cascades. Matta's theory applies to several cascades, but is presumes that the overall axial length of the cascades is much less than the wavelength. However, this prerequisite generally is not met in practice. Even with the relatively low-frequency combustion chamber noise, the distance between the combustion chamber and the low-pressure turbine outlet is the equivalent of half to total wavelength.

The objective of the paper is to describe a theory according to which the sound propagation via several rotors and stators, arranged in series, can be calculated even with wavelengths that are less than the axial length of a multistage turbomachine. Applied in conjunction with an appropriate method for simulating the noise emitted by the combustion chamber and the rotors and stators in a turbomachine, the new theory permits precise calculation of the combustion chamber noise as well as the noise of a multistage turbomachine.

Concept of the Theory

In the following theory, first of all, the sound propagation through the individual components of an axial-flow turbomachine is determined. As illustrated in Fig. 1, these are the rotors, stators, bladeless annuli between the rotors and stators, as well as the nozzle or intake. The sound propagation through the complete turbomachine is then determined from the parameters which describe the sound propagation through the components.

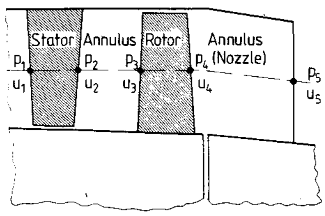

Fig. 1 Elements of an axial turbomachine through which sound propagates.

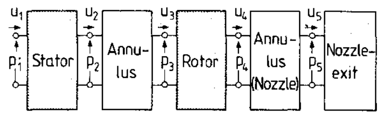

Fig. 2 Electroacoustic analogy for sound propagation in a turbine.

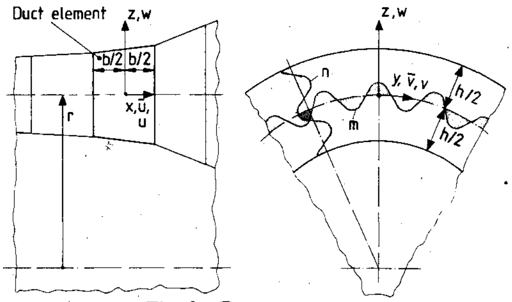

Fig. 3 Duct geometry.

The theory is based on the assumption that the flow through the components is isentropic and compressible. It is further assumed that these components are coupled acoustically primarily via their far fields. The interaction of the components via their near fields and the vorticity waves generated by the cascades are, therefore, ignored.

Vorticity waves occur when sound strikes a cascade. They represent a pure velocity disturbance, which is transmitted by the main flow and which can be turned back into sound waves on striking a cascade. As will be shown later, the recovery of sound energy from vorticity energy, however, is not very great.

It is a further prerequisite of the following theory that the plane waves and the modes circulating in the annulus propagate through the turbomachine without modal distortions. As cross-sectional changes in ducts are usually very small for aerodynamic reasons, modal distortions may be expected to occur predominantly at the cascades. A modal distortion then occurs when the wavelength is smaller than about twice the distance between two blades. Therefore, the method applies only up to this wavelength.

Subject to the prerequisites just given, the sound propagation through each annulus and the cascades can be quite formally described by the following matrix equation:

$$\begin{pmatrix} p \\ u \end{pmatrix}_{j-1} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{pmatrix} p \\ u \end{pmatrix}_j \tag{1}$$

which provides a relationship between the sound fields at both sides of an annulus or cascade.

As the electrical circuit elements with two input and two output terminals or poles can be represented mathematically in a similar manner, Eq. (1) is described as a four-pole equation. Because of this mathematical similarity, the sound propagation through a multistage turbomachine may also be described by an electrical analogy in which electrical four-poles are allocated to the annulus elements and cascades. As illustrated in Fig. 2, a turbomachine's equivalent circuit is, therefore, a four-pole chain. The terminal of this chain is formed by a so-called two pole, which simulates the impedance of the nozzle opening or the intake in the case of a compressor. As the equivalent circuit shows, all the elements are interconnected in such a way that the sound pressure and the axial components of the acoustic velocity are always equal at corresponding junctions.

Description of the sound propagation properties of the turbomachine components by means of four-poles is not merely highly graphic, but also very practical mathematically. If the four-pole matrices of a four-pole chain are known, a four-pole matrix describing the complete chain can be calculated by simple matrix multiplication, where the coupling conditions mentioned between the four poles are automatically taken into consideration

$$\begin{pmatrix} p \\ u \end{pmatrix}_1 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{12} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{23} \cdots \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{(j-1)j} \begin{pmatrix} p \\ u \end{pmatrix}_j$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{1j} \begin{pmatrix} p \\ u \end{pmatrix}_j \tag{2}$$

From this matrix and the chain-terminating impedance $Z_j = p_j/u_j$ the transmission coefficient T of the chain can be evaluated

$$T = \frac{p_j}{p_1} = \frac{1}{A_{1j} + B_{1j}/Z_j} \tag{3}$$

Furthermore, it is easy to transform the terminating impedance to the beginning of the chain and to determine the reflection coefficient R from it.

$$Z_1 = \frac{p_1}{u_1} = \frac{A_{1j} + B_{1j}/Z_j}{C_{1j} + D_{1j}/Z_j}, \quad R = \frac{Z_1 - Z_0}{Z_1 + Z_0} \tag{4}$$

There now follows an explanation of the determination of the four-pole matrices belonging to the bladeless annular ducts and cascades. Also, instructions will be given how to compute the impedance of the nozzle and the intake.

Four-Pole Matrix for Annular Duct

Figure 3 shows the model on which the calculation of the sound propagation through an annular duct or annular duct four-pole matrix is based. The annular duct has a constant mean radius r and is allowed to expand or contract arbitrarily in the axial direction. Flow through it occurs both in the axial and the circumferential directions, where the axial flow velocity $\bar{u}$ changes as a result of the change in cross section through the axis of the duct. However, $\bar{u}$ ought to be constant over the height of the duct.

It is further assumed that the duct height h is appreciably less than the radius r. In this case, the curvature of the duct cross section has no influence on the flow and sound field, i.e., the velocity component $\bar{v}$ is constant above the height of the duct, and the flow is, therefore, irrotational. Furthermore, the sound modes m and n are orthogonal.

There are, at present, no sound-propagation theories available for such ducts. The existing theories ignore either the swirling flow component or the change in the duct's cross section. However, both characteristics are typical of turbomachines, and must be taken into consideration when calculating the sound propagation. For instance, the change in the duct's cross section has a decisive influence on the propagation of the low-frequency combustion chamber noise. The swirl, above all, greatly affects the propagation of the higher modes.

Calculation of the sound propagation in a noncylindrical duct with a swirling flow is rather difficult, especially because the wave numbers and the coefficients of the wave equation, occurring in the duct, vary with the coordinates in the axial direction of the duct. Therefore, only a numerical method may be considered for calculating the sound propagation. In the present case the following method was chosen.

First, the annular duct is divided into small elements, within which the change in coefficients can be ignored. The sound field in the duct elements is then calculated by a potential flow formulation of the wave equation and the assumption of constant coefficients in this equation. Next, four-pole matrices, whose multiplication finally provides a four-pole matrix describing the whole duct, are determined from the sound-field equations of the individual duct elements.

Sound Field in a Duct Element

The potential flow approach of the sound field in the duct is based on the continuity equation ensuing. For a duct with variable cross section the equation is as follows.

$$\frac{1}{F}\frac{\partial}{\partial x}(PUF)+\frac{\partial}{\partial y}(PV)+\frac{\partial}{\partial z}(PW)+\frac{\partial}{\partial t}P=0 \tag{5}$$

In this equation we can separate the density P and the velocity components U, V, and W into steady and acoustic quantities. Thereby, bearing in mind the already given assumptions concerning the duct geometry and flow, we obtain the following equations.

$$P=\bar{\rho}(x)+\rho(x,y,z,t),\quad U=\bar{u}(x)+u(x,y,z,t)$$

$$V=\bar{v}+v(x,y,z,t),\quad W=\bar{w}+w(x,y,z,t),\quad \bar{w}=0 \tag{6}$$

Substitution of Eqs. (6) into Eq. (5) gives two continuity equations, one for the steady quantities, and the other for the acoustic quantities. However, in the following, we are only interested in the equation for the acoustic quantities, as the steady flow can be determined simply by using the one-dimensional isentropic gasdynamic relations.

In conjunction with the linearized isentropic equation

$$\rho=\frac{1}{c^2}p \tag{7}$$

and the known relations for the velocity potential ϕ

$$u=\frac{\partial\phi}{\partial x},\quad v=\frac{\partial\phi}{\partial y},\quad w=\frac{\partial\phi}{\partial z},\quad p=-\bar{\rho}\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}+\bar{v}\frac{\partial}{\partial y}\right)\phi \tag{8}$$

the continuity equation for the acoustic quantities provides the wave equation for the sound propagation in the duct described earlier. If second-order terms are ignored, the wave equation has the following form:

$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}+\frac{E_a}{c}\frac{\partial\phi}{\partial t}+2\frac{Ma_x}{c}\frac{\partial^2\phi}{\partial x\partial t}+2\frac{Ma_y}{c}\frac{\partial^2\phi}{\partial t\partial y}$$
$$+(Ma_x^2-1)\frac{\partial^2\phi}{\partial x^2}+E_b\frac{\partial\phi}{\partial x}+(Ma_y^2-1)\frac{\partial^2\phi}{\partial y^2}$$
$$+E_c\frac{\partial\phi}{\partial y}-\frac{\partial^2\phi}{\partial z^2}+2Ma_xMa_y\frac{\partial^2\phi}{\partial x\partial y}=0 \tag{9}$$

with

$$E_a=Ma_x\left(\frac{\partial\ell n\bar{u}}{\partial x}+\frac{\partial\ell nF}{\partial x}-2\frac{\partial\ell nc}{\partial x}+\frac{\partial\ell n\bar{\rho}}{\partial x}\right)$$

$$E_b=Ma_x^2\left(2\frac{\partial\ell n\bar{u}}{\partial x}+\frac{\partial\ell nF}{\partial x}-2\frac{\partial\ell nc}{\partial x}+\frac{\partial\ell n\bar{\rho}}{\partial x}\right)-\frac{\partial\ell n\bar{\rho}}{\partial x}-\frac{\partial\ell nF}{\partial x}$$

and

$$E_c=Ma_xMa_y\left(\frac{\partial\ell n\bar{u}}{\partial x}+\frac{\partial\ell nF}{\partial x}-2\frac{\partial\ell nc}{\partial x}+\frac{\partial\ell n\bar{\rho}}{\partial x}\right)$$

Because of the dependence of its coefficients on x, Eq. (9) cannot be integrated completely analytically. Nevertheless, under the assumption that the coefficients within a duct element of width b (Fig. 3) are constant or vary only slightly, Eq. (9) can be integrated analytically in sections by assuming the integral to be an exponential expression [see Eq. (11)]. As already mentioned, the section integrals can then be coupled using the four-pole theory.

With this sectionwise integration it is logical to allocate to the coefficients in Eq. (9) values that are representative of the duct element in question. Several possibilities present themselves for determining these values. Good results are supplied by a method in which Ma_x, Ma_y, and c are each substituted by a mean value from the values for $x=-b/2$ and $b/2$, and the derivatives in E_a, E_b, and E_c are approximated as follows.

$$\frac{\partial\ell n g(x)}{\partial x}\simeq\frac{1}{g}\frac{\Delta g}{\Delta x}=\frac{2}{b}\frac{g(x=b/2)-g(x=-b/2)}{g(x=b/2)+g(x=-b/2)} \tag{10}$$

In this connection it also should be noted that the necessary division of the duct into several elements depends solely on the variation in the coefficients with x. Increasing discretization with the frequency, as required by several numerical methods, is not necessary here.

The sectionwise integration of Eq. (9) should be based on the two particular solutions

$$\phi^{\pm}=\cos[k_z(h/2+z)]\exp[i(\omega t\mp k_x^{\pm}x-k_yy)] \tag{11}$$

Here, the potential ϕ^+ describes a wave propagating in the x direction. The wave belonging to ϕ^- propagates in the opposite direction. Both waves run with the same velocity in the y direction and also have the same sound-pressure distribution along z.

The propagation constants k_y and k_z in Eq. (11) are specified by boundary conditions. From the condition that the circumference of the annular duct must be equal to an integer multiple of the wavelength in the y direction, it follows that

$$k_y=m/r\ \text{with}\ m=\ldots-2,\ -1,\ 0,\ 1,\ 2\ldots \tag{12}$$

where negative m's belong to waves running opposite to the y direction. Further, by virtue of the fact that the annular duct has hard walls, pressure maximums must lie at $z=-h/2$ and $h/2$. Accordingly,

$$k_z=n\pi/h\ \text{with}\ n=0,\ 1,\ 2\ldots \tag{13}$$

applies.

The two unknown propagation constants k_x^+ and k_x^- are determined finally by substituting Eq. (11) into Eq. (9). One thus obtains

$$k_x^{\pm}=\mp\alpha+\beta \tag{14}$$

with

$$\alpha=\frac{1}{1-Ma_x^2}\left(Ma_xk_0-Ma_xMa_yk_y-\frac{i}{2}E_b\right)$$

and

$$\beta=\frac{1}{1-Ma_x^2}\Big[\Big(Ma_xk_0-Ma_xMa_yk_y-\frac{i}{2}E_b\Big)^2$$
$$-(1-Ma_x^2)\,(ik_0E_a-k_0^2+2Ma_yk_0k_y$$
$$+(1-Ma_y^2)\,k_y^2-ik_yE_c+k_z^2)\Big]^{1/2}$$

Bearing in mind that a linear combination of particular solutions to a partial differential equation also represents a solution, with

$$\phi=a^+\phi^+ + a^-\phi^- \tag{15}$$

one obtains a new general solution to Eq. (9). Equation (15) now describes the potential of a sound field that is composed of one wave running in the x direction and of another running against the x direction, where a^+ and a^- are factors which can be determined from further boundary conditions of the sound field. Using Eqs. (8) and (11-14), the sound field in an annular duct element can now be determined from Eq. (15). For the acoustic velocity component in the x direction and the sound pressure, this gives rise to the following equations.

$$u=i\cos(k_z(h/2+z))\,e^{i(\omega t-k_yy)}\,[(\alpha-\beta)\,a^+e^{i(\alpha-\beta)x}$$
$$+(\alpha+\beta)\,a^-e^{i(\alpha+\beta)x}]$$
$$p=-i\bar{\rho}\cos(k_z(h/2+z))\,e^{i(\omega t-k_yy)}\,[(\omega+\bar{u}(\alpha-\beta)$$
$$-\bar{v}k_y)\,a^+e^{i(\alpha-\beta)x}+(\omega+\bar{u}(\alpha+\beta)-\bar{v}k_y)\,a^-e^{i(\alpha+\beta)x}] \tag{16}$$

Four-Pole Matrix of a Duct Element

The desired four-pole matrix ought to provide a relationship between the sound pressures p and the acoustic velocities u in the planes $x=-b/2$ and $b/2$. If the sound field in the plane $x=-b/2$ is identified by the index 1 and that in the plane $x=b/2$ by the index 2, on the basis of Eq. (1), the following equations for the matrix elements are obtained.

$$A=\frac{p_1}{p_2}\Big|_{u_2=0},\quad B=\frac{p_1}{u_2}\Big|_{p_2=0},\quad C=\frac{u_1}{p_2}\Big|_{u_2=0},\quad D=\frac{u_1}{u_2}\Big|_{p_2=0} \tag{17}$$

In this, p_1 and u_1 as well as p_2 and u_2 are derived from Eqs. (16), namely, by calculating these equations once for $x=-b/2$ and once for $x=b/2$. Substitution of these p and u values into Eqs. (17) and elimination of a^+ and a^-, gives the following equations for the matrix elements:

$$A=\Big(\gamma_1\frac{\eta_2}{\epsilon_2}-\delta_1\Big)\Big/\Big(\gamma_2\frac{\eta_2}{\epsilon_2}-\delta_2\Big)$$

$$B=\Big(\gamma_1\frac{\delta_2}{\gamma_2}-\delta_1\Big)\Big/\Big(\epsilon_2\frac{\delta_2}{\gamma_2}-\eta_2\Big)$$

$$C=\Big(\epsilon_1\frac{\eta_2}{\epsilon_2}-\eta_1\Big)\Big/\Big(\gamma_2\frac{\eta_2}{\epsilon_2}-\delta_2\Big)$$

$$D=\Big(\epsilon_1\frac{\delta_2}{\gamma_2}-\eta_1\Big)\Big/\Big(\epsilon_2\frac{\delta_2}{\gamma_2}-\eta_2\Big) \tag{18}$$

with the abbreviations:

$$\gamma_1=-i\bar{\rho}_1(\omega+\bar{u}_1(\alpha-\beta)-\bar{v}k_y)\,e^{-i(\alpha-\beta)b/2}$$
$$\delta_1=-i\bar{\rho}_1(\omega+\bar{u}_1(\alpha+\beta)-\bar{v}k_y)\,e^{-i(\alpha+\beta)b/2}$$
$$\gamma_2=-i\bar{\rho}_2(\omega+\bar{u}_2(\alpha-\beta)-\bar{v}k_y)\,e^{i(\alpha-\beta)b/2}$$
$$\delta_2=-i\bar{\rho}_2(\omega+\bar{u}_2(\alpha+\beta)-\bar{v}k_y)\,e^{i(\alpha+\beta)b/2}$$
$$\epsilon_1=i(\alpha-\beta)e^{-i(\alpha-\beta)b/2},\quad \eta_1=i(\alpha+\beta)e^{-i(\alpha+\beta)b/2}$$
$$\epsilon_2=i(\alpha-\beta)e^{i(\alpha-\beta)b/2},\quad \eta_2=i(\alpha+\beta)e^{i(\alpha+\beta)b/2}$$

It remains to be stated that in the case of plane waves and nonswirling flow, this method can also be used for calculating the sound propagation through convergent and divergent ducts regardless of the cross-sectional shape.

Four-Pole Matrix for Cascades

The cascade four-pole matrices can be determined from the reflection and transmission coefficients of the cascade. There are several methods[3-7] available for calculating these coefficients, where that of Koch[3] is most generally applicable.

Figure 4 shows the model on which this theory is based. The cascade consists of plates of the length l and thickness zero. It has the pitch s, the stagger angle β_s, and flow occurs in the direction of the plates at a specific Mach number. The cascade is struck by a plane wave, identified by its frequency and angle of incidence. With wavelengths of about two pitches, the wave in question will be split up into several transmitted and reflected waves by diffraction, similar to an optical diffraction grating. However, for our purposes we are only interested in the wavelength range in which this diffraction does not occur. Several transmitted and reflected waves should indicate a modal distortion, which, as already mentioned, is incompatible with the prerequisites of the four-pole theory.

Evaluating the reflection and transmission coefficients of a cascade, the sound-incidence angle, defined in Fig. 4, is determined from the propagation constants k_x, k_y, and k_z of the duct element containing the cascade. When determining the propagation of sound through rotors, it must be remembered that the coordinate system is associated with the cascade, and thus moves with the cascade. As the sound in a moving coordinate system has a different frequency to that in a stationary system, the rotor calculation requires frequency transformation.

Calculation of the cascade four-pole matrix of a duct mode requires its reflection and transmission coefficients for the sound propagation, both in and against the direction of flow. With the aid of these coefficients four equations can be formulated, from which, in turn, the four complex elements of the cascade matrix can be determined.

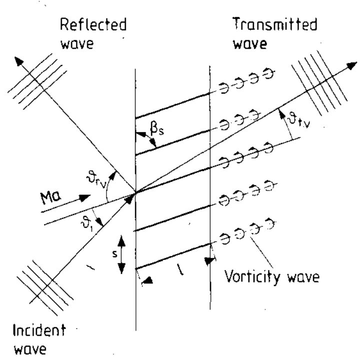

Fig. 4 Blade row geometry.

In the following determination of the matrix elements, the sound pressures and acoustic velocities at the leading edge of the cascade are identified by the index *1*, and those at the trailing edge of the cascade by the index *2*. Further, for differentiating the direction of the sound propagation through the cascade, the values belonging to the downstream propagation are allocated a plus sign again. The values associated with the upstream sound propagation, in contrast, bear a minus sign.

If it is now assumed that the sound impinges in the downstream direction onto the reflection-free terminated cascade, from its transmission and reflection coefficients T^+ and R^+ and its termination impedance Z_0^+, the following equations can be derived for the sound pressures and acoustic velocities at both sides of the cascade:

$$\frac{p_2^+}{p_1^+} = \frac{T^+}{1+R^+}, \quad \frac{p_1^+}{u_1^+} = Z_1^+ = Z_0^+\frac{1+R^+}{1-R^+}, \quad \frac{p_2^+}{u_2^+} = Z_0^+ \tag{19}$$

where Z_0^+ results from Eq. (16) for $a^- = 0$ as

$$Z_0^+ = \bar{\rho} c \frac{Ma_y k_y - Ma_x(\alpha-\beta) - k_0}{\alpha-\beta} \tag{20}$$

Analogous to the preceding equations, the following relations apply for the upstream sound propagation through the cascade:

$$\frac{p_1^-}{p_2^-} = \frac{T^-}{1+R^-}, \quad \frac{p_2^-}{u_2^-} = Z_2^- = Z_0^-\frac{1+R^-}{1-R^-}, \quad \frac{p_1^-}{u_1^-} = Z_0^- \tag{21}$$

with

$$Z_0^- = \bar{\rho} c \frac{Ma_y k_y - Ma_x(\alpha+\beta) - k_0}{\alpha+\beta} \tag{22}$$

As the cascade four-pole matrix is to describe the sound propagation both in the direction of flow and against it, this matrix must satisfy the equations

$$\begin{pmatrix} p_1^+ \\ u_1^+ \end{pmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{pmatrix} p_2^+ \\ u_2^+ \end{pmatrix} \text{ and } \begin{pmatrix} p_1^- \\ u_1^- \end{pmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{pmatrix} p_2^- \\ u_2^- \end{pmatrix} \tag{23}$$

By substituting these equations into Eqs. (19) and (21) one now obtains the four equations mentioned for determining the matrix elements.

$$AZ_0^+ + B - CZ_0^+Z_1^+ - DZ_1^+ = 0, \quad A + B/Z_0^+ = (1+R^+)/T^+$$

$$AZ_2^- + B - CZ_0^-Z_2^- - DZ_0^- = 0, \quad A + B/Z_2^- = T^-/(1+R^-) \tag{24}$$

The solution of this system of equations finally provides

$$A = \frac{T^+T^-Z_2^- - (1+R^+)(1+R^-)Z_0^+}{T^+(1+R^-)(Z_2^- - Z_0^+)}$$

$$B = Z_0^+Z_2^- \frac{(1+R^+)(1+R^-) - T^+T^-}{T^+(1+R^-)(Z_2^- - Z_0^+)}$$

$$C = \frac{T^+T^-Z_1^+Z_2^- - Z_0^+Z_0^-(1+R^+)(1+R^-)}{Z_0^-Z_1^+T^+(1+R^-)(Z_2^- - Z_0^+)}$$

$$D = Z_0^+Z_2^- \frac{Z_0^-(1+R^+)(1+R^-) - T^+T^-Z_1^+}{Z_0^-Z_1^+T^+(1+R^-)(Z_2^- - Z_0^+)} \tag{25}$$

for the individual matrix elements.

Impedance of Nozzle and Intake

If the sound waves propagate through the end of the nozzle or the intake without reflection, the previously given theory already can be used to determine the sound propagation through the complete turbomachine. In this, the nozzle and intake openings have the same impedance as the waves which would propagate free of reflection in hypothetical cylindrical ducts connected to the nozzle or intake openings. Hence, the nozzle and intake impedances can be calculated with the aid of Eqs. (20) and (22).

Because, in practice, only the sound propagation of cuton modes is of interest and these propagate largely free of reflection in and through the nozzle and intake, generally speaking, the sound propagation calculation requires only the impedance of plane waves to be determined. For this, the methods according to Munt[10] and Lansing et al.,[11] which take into consideration the influence of the flow on the propagation of the sound, can be used for nozzles and intakes of circular cross section. The impedance of nozzles and intakes of annular cross section can be determined by the method of Zorumski.[12] However, it must be admitted that this method pays no attention to the influence of the flow on the sound propagation. Also, this theory is based on an annular duct fitted with a flange.

Verification of the Theory

For verifying the theory, comparison was made with measurements and results from different theories. Thereby, the present theory was proved by results on sound propagation in nonuniform ducts of circular cross section and annular ducts containing cascades. Verification for nonuniform annular ducts without cascades was not possible, as there were no suitable results available. In the following some of the comparisons with other theoretical results and experimental data will be illustrated.

Figure 5 shows the resistance and reactance spectrum of a reflection-free terminated exponential horn, calculated once according to the known theory of Webster and once according to the four-pole theory. Despite the fact that the horn was split into just four elements, the four-pole theory corresponds well with Webster's analytical theory.

In order to prove the complete theory, i.e., the theory of sound propagation through a series of cascades arranged one behind the other, results obtained by Amiet[8] and measurements by Doyle and Matta[2] were used.

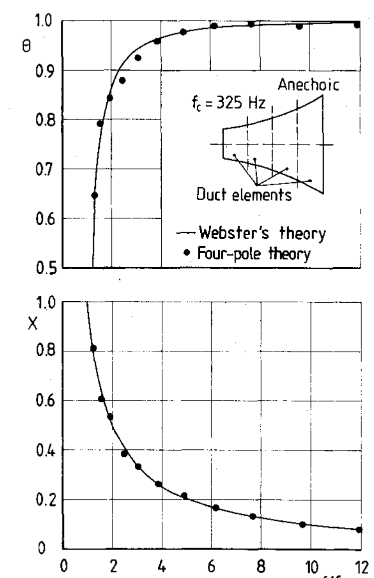

Fig. 5 Specific resistance and reactance of an exponential horn, comparison between four-pole and Webster's theory.

Figure 6 shows the transmission coefficient in the propagation of sound through a rotor and stator in relationship to the angle of incidence, calculated once according to the four-pole theory and once according to Amiet's theory[8] for the propagation of sound through two cascades. The interesting point of this comparison is the fact that Amiet's theory bears in mind the interaction of the cascades as a result of the vorticity waves, whereas they are ignored in the four-pole theory. In the present case, the interaction consists in the fact that the vorticity waves emanating from the stator strike the rotor. The small difference in the results shows that the effect of this interaction on the sound propagation is apparently slight.

In the measurements of Doyle and Matta, the sound propagation through a three-stage turbine was determined. Figure 7 shows a longitudinal section through the turbine, in which several sound-measuring probes are arranged circumferentially around the walls at the inlet and outlet sides, i.e., in planes 1 and 2. When the turbine was exposed to sound waves in the direction of flow, these probes were used for measuring the mean difference in the sound pressure level between the two planes. The sound source in this case was a siren.

Theory and measurements were compared under the assumption that in plane 1 the cuton modes are all excited with the same amplitude. In calculating the impedance in plane 3 it was further assumed that the annular duct is flanged to a larger chamber at this point.

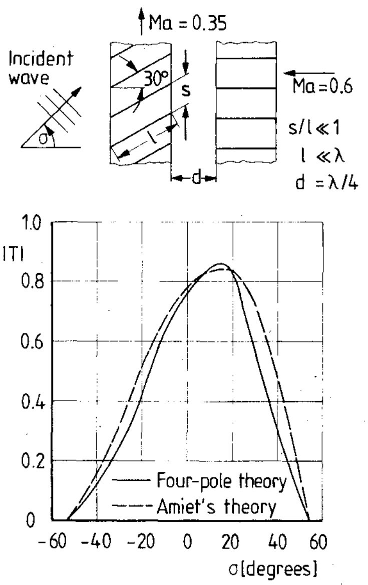

Fig. 6 Sound pressure transmission coefficient of a turbine stage, comparison between four-pole and Amiet's theory.[8]

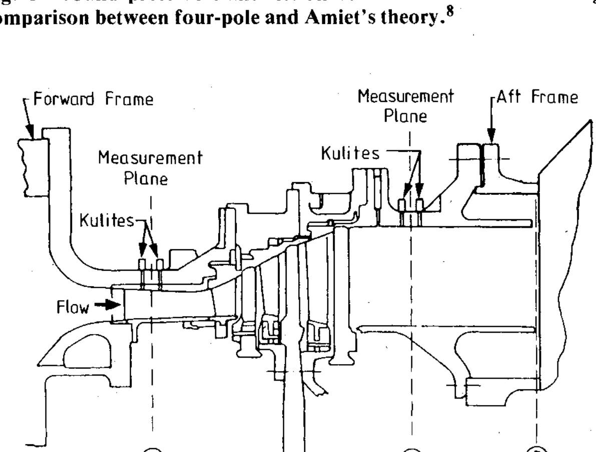

Fig. 7 Turbine rig for sound transmission measurements.[2]

Figure 8 shows the measured and calculated mean sound pressure level difference between the turbine inlet and outlet as a function of the frequency. Furthermore, the cuton mode range is given for each measured point in this diagram. From this it should be understood that up to approximately 250 Hz only plane waves and from then on also higher modes can propagate. A comparison between the theory and measurements shows that the theory provides good results for both frequency ranges.

Results of Parameter Studies

The new method was used for investigating the influence of various geometric and aerodynamic parameters on the propagation of sound in noncylindrical annular ducts as met in turbomachines. In this, in contrast to earlier investigations, it was borne in mind that the flow through these ducts, in general, has a swirling component. In addition, the sound propagation in various multistage turbomachines was studied.

When investigating the propagation of sound in annular ducts, the sound-pressure reflection coefficient of the plane wave ($m=0$) and of the first two circumferential modes ($m=1$ and -1) at the duct inlet was determined. This calculation was made under the assumption that a cylindrical annular duct with a reflection-free end forms the continuation of the duct outlet. In detail, the following parameters were varied in this investigation: Mach number in the circumferential and axial directions, ratio between the cross-sectional areas of the duct outlet and inlet, and also the duct length.

Figure 9a illustrates the effect of the circumferential Mach number Ma_y on the sound propagation with constant axial Mach number Ma_x, where the Mach numbers in the diagram refer to the duct inlet. In accordance with expectations, the circumferential Mach number has no effect on the reflection coefficient of the plane waves ($m=0$). In the case of the circumferential modes 1 and -1, however, an increase in Ma_y causes the reflection-coefficient spectrum to shift to lower or higher frequencies, where the shift of the -1 mode is the stronger. With Ma_y the cutoff frequencies, given in Fig. 9a for the two modes, also change. Below these frequencies the propagation of the modes is attenuated in the upstream cylindrical annular duct. On the whole, Fig. 9a shows that swirl influences the propagation of higher modes very markedly.

The influence exerted by the axial Mach number at the duct inlet on the sound propagation in a divergent annular duct, in which swirl occurs, is shown in Fig. 9b. Increasing this Mach number from 0.3 to 0.6 causes a considerable increase in all reflection coefficients. Furthermore, in this case a shift, even though it be small, in all spectra toward lower frequencies is observable.

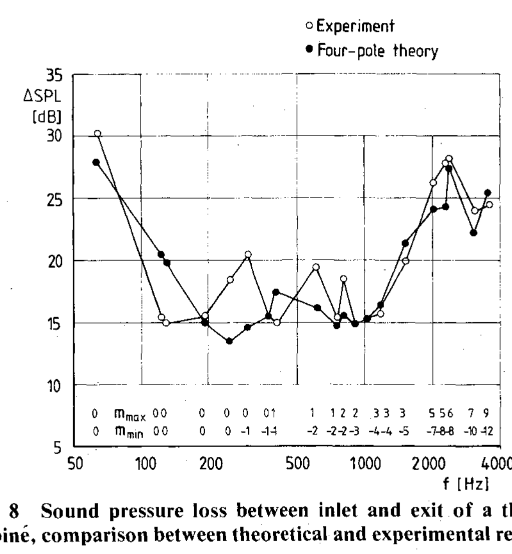

Fig. 8 Sound pressure loss between inlet and exit of a three-stage turbine, comparison between theoretical and experimental results.[2]

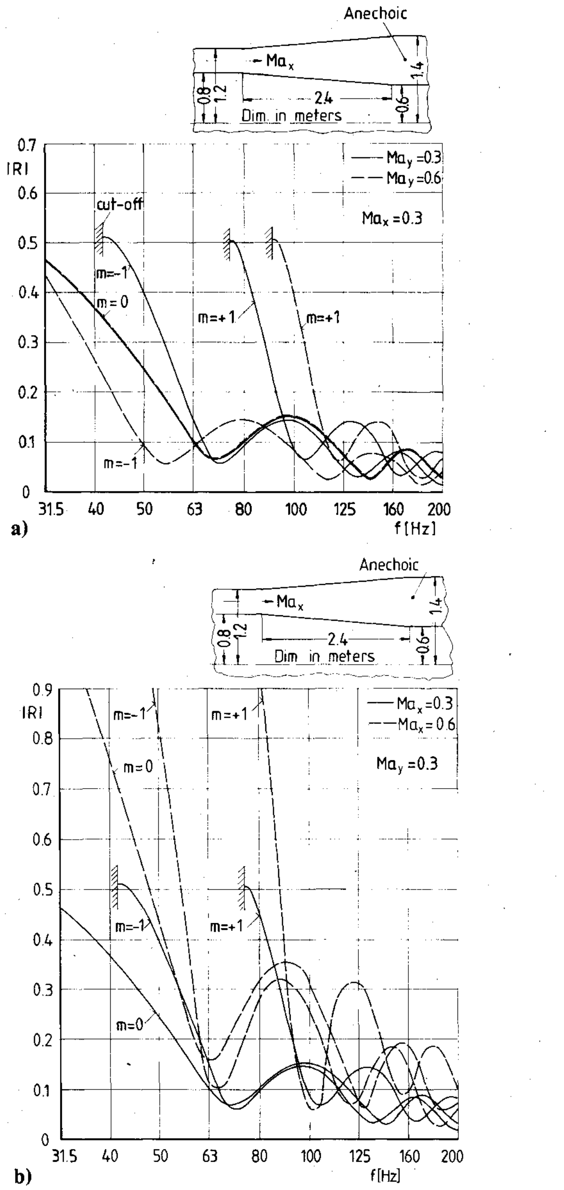

Fig. 9 Sound propagation in a divergent annulus with a swirling mean flow; a) influence of circumferential Mach number on reflection coefficient and b) influence of axial Mach number on reflection coefficient.

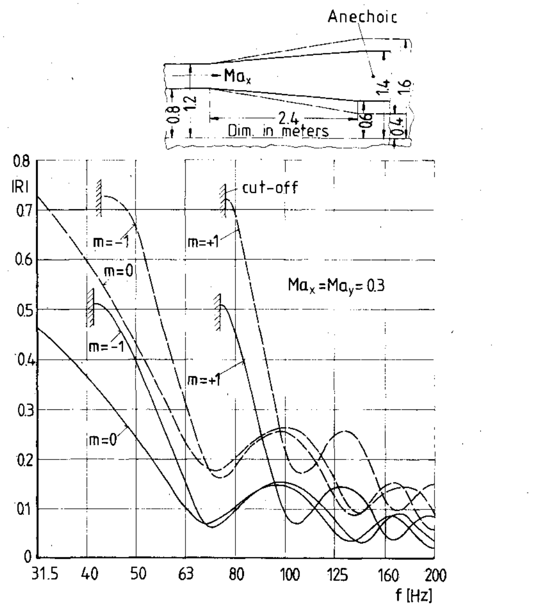

Fig. 10 Divergent annulus with a swirling mean flow; influence of area ratio on reflection coefficient.

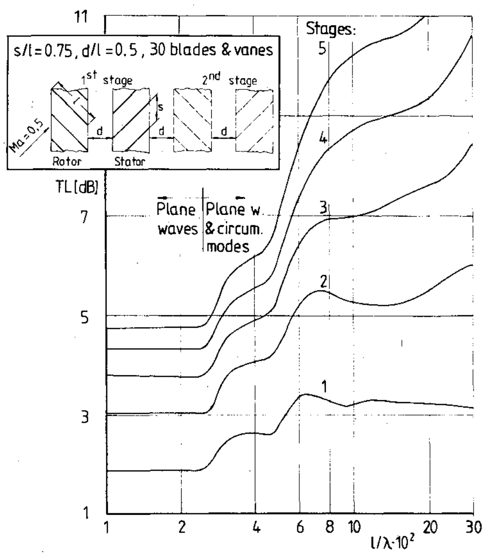

Fig. 11 Multistage turbines; influence of stage number on transmission loss.

As illustrated in Fig. 10, an increase in the area ratio results in greater reflection of all modes. The resonance frequencies identified in the spectra by lower reflection coefficients remain unaffected by the area ratio. On the other hand, an increase in the duct length causes a reduction in these frequencies and has only a minior effect on the level of the reflection factors (not illustrated by a figure).

The investigation into the propagation of sound through multistage turbomachines included calculations of the transmission loss TL (difference between power level of incidental and transmitted sound) in single- to five-stage turbines. For simplicity's sake, it was assumed here that the turbines have cylindrical annular ducts and that the same stages are repeated several times, one behind the other, in the multistage turbines. Disregarding the duct expansion in the direction of flow means, however, that in reality the transmission losses in turbines with similar cascades are somewhat greater. It was further assumed in this investigation that the turbine outlets have an anechoic termination.

The transmission losses of the individual turbines are plotted in Fig. 11 against the ratio between the blade chordal length and the wavelength. We are dealing here with a transmission loss averaged over all cuton modes, that was determined under the assumption that at the turbine inlet all incident modes carry the same acoustic power. Figure 11 shows that the transmission loss increases clearly in all turbines as soon as higher duct modes ($m \neq 0$) can propagate in the turbines in addition to the plane waves. Further, the transmission loss becomes greater, the greater the number of the turbine stages. Figure 11 also shows that the increase in the transmission loss with increasing number of stages is markedly nonlinear, especially with the greater wavelengths. This may be attributed to strong coupling between the individual stages or cascades. The transmission loss in a multistage turbine, therefore, cannot be determined by summing up the transmission losses of the individual stages.

Summary and Conclusions

A method has been developed for calculating the propagation of sound in multistage turbomachines. Amongst other things, this method bears in mind that turbomachines generally have nonuniform annular ducts and a mean flow with a swirling component. Subject to large hub to tip radius ratios, the new method can be used for calculating the propagation of higher modes, in addition to that of plane waves.

The method is based on the so-called four-pole theory, in which the acoustic properties of bladeless annular ducts and cascades are described by complex 4×4 matrices. This theory implies that the sound propagation in turbomachines occurs without modal distortions. The more extensive modal distortions, however, are only to be expected from frequencies whose wavelength is smaller than about two blade pitches. Comparison with measurements and the results of calculations from other theories proved good compatibility.

In a parameter study it is shown that acoustic coupling of the cascades, especially with the lower frequencies, is very pronounced. Therefore, no conclusions can be drawn about the transmission loss in a multistage turbomachine from the sound power transmission losses in individual cascades. It is further shown that the sound propagation in the ducts is affected not just by their geometric parameters, but also to an extensive degree by the swirling flow component.

Acknowledgment

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References

[1]Vaidya, P. G. and Dean, P. D., "State of the Art of Duct Acoustics," AIAA Paper 77-1279, 1977.

[2]Doyle, V. L. and Matta, R. K., "Attenuation of Upstream-Generated Low Frequency Noise by Gas Turbines," NASA CR-135219, 1977.

[3]Koch, W., "On the Transmission of Sound Waves through a Blade Row," *Journal of Sound and Vibration*, Vol. 18, No. 1, 1971, pp. 111-128,

[4]Kaji, S. and Okazaki, T., "Propagation of Sound Waves through a Blade Row. II: Analysis on the Acceleration Potential Method," *Journal of Sound and Vibration*, Vol. 11, No. 3, 1970, pp. 355-375.

[5]Amiet, R. K., "Transmission and Reflection of Sound by a Blade Row," AIAA Paper 71-181, 1971.

[6]Mani, R. and Horvay, G., "Sound Transmission through Blade Rows," *Journal of Sound and Vibration*, Vol. 12, No. 1, 1970, pp. 59-83.

[7]Bekofske, K., "Attenuation of Acoustic Energy across a Turbine Stage with Subsonic Relative Flow," ASME Paper 75-GT-17, 1975.

[8]Amiet, R. K., "Transmission and Reflection of Sound by Two Blade Rows," *Journal of Sound and Vibration*, Vol. 34, No. 3, 1974, pp. 399-412.

[9]Matta, R. K. and Mani, R., "Theory of Low-Frequency Noise Transmission through Turbines," NASA CR-159457, 1979.

[10]Munt, R. M., "Acoustic Transmission Properties of a Jet Pipe with Subsonic Jet Flow," Part I: "The Cold Jet Reflection Coefficient," Part II: "The Cold Jet Radiated Power," Mathematics Dept., University of Dundee, Scotland, DD14HN, Sept. 1978.

[11]Lansing, D. L., Drischler, J. A., and Pusey, C. G., "Radiation of Sound from an Unflanged Circular Duct with Flow," Paper presented at the 79th Meeting of the Acoustical Society of America, 1970.

[12]Zorumski, W. E., "Generalized Radiation Impedances and Reflection Coefficients of Circular and Annular Ducts," *Journal of the Acoustical Society of America*, Vol. 54, No. 6, 1973, pp. 1667-1673.